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THE APSIDAL-MOTION TEST FOR MODELS OF MAIN-SEQUENCE STARS

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ABSTRACT

Apsidal-motion constants, k_2, have been determined for models of homogeneous main-sequence stars constructed with the new "Thomas-Fermi" opacities of Carson. These models are similar to those constructed with the "hydrogenic" opacities of Cox and Stewart up to $\sim 7\, M_\odot$, but yield smaller values of k_2 at higher masses. Nevertheless, the new theoretical constants are still systematically too large compared with most of the empirical constants known for members of well-observed binary systems. Uniform rotation is found to lower the theoretical constants only slightly. The effect of normal evolution during core hydrogen burning, however, produces good detailed agreement with observations. The primary of α Vir is probably evolving normally if Carson's opacities are adopted.

Subject headings: apsidal motion — binaries — interiors, stellar — rotation, stellar — stars, individual — stellar evolution

I. INTRODUCTION

The line of apsides in a close binary system precesses at a rate governed by the density distribution within the two components. The apsidal-motion constant k_2, corresponding to the second harmonic of the mutual tidal distortion (of chief importance in practice), has been frequently computed for theoretical stellar models and compared with observations, most recently by Kushwaha (1957), Schwarzschild (1958), Kopal (1965), Mathis (1967), Semeniuk and Paczynski (1968), Petty (1973), Mathis and Odell (1973), and Odell (1974). The pre–1966 models, constructed with a modified Kramers law representing the opacity due to atomic absorption, gave values of k_2 that were systematically too large compared with the observed values, by a factor of about 3. The later models, based on the Cox-Stewart opacities, reduced the discrepancy but did not eliminate it. Since comparisons were made with supposedly unevolved main-sequence stars, part of the discrepancy was assigned to evolutionary effects (which reduce k_2). However, suspicion was also placed on the adopted opacities and the adopted initial chemical composition, as well as on the observed values of k_2.

In the present work, the new Carson (1974) opacities will be employed in stellar models to recalculate k_2 for both unevolved and evolved main-sequence stars. These opacities are based on the hot Thomas-Fermi atomic model for all elements heavier than hydrogen and helium. They differ significantly from the earlier, less refined opacities published, for a few temperatures and densities, by Carson, Mayers, and Stibbs (1968) on the basis of the same atomic model; and they now agree closely with the "hydrogenic" opacities of Cox and Stewart (1965) under the temperature and density conditions of the Sun. Not unexpectedly, they begin to deviate from the Cox-Stewart opacities when the density becomes high (for the new opacities, higher than solar densities). However, they also deviate at very low densities, in the sense that Carson's opacities are larger than those of Cox and Stewart. As we shall see, this has a marked effect on k_2 at high stellar masses.

II. THEORETICAL MODELS

Homogeneous stellar models have been constructed with the same input physics as was used by Stothers (1972), except for the adoption here of Carson's new opacity tables. Where necessary, linear interpolation in the tables (with all quantities expressed in logarithmic form) has been employed. As in the previous work, a simple model atmosphere has been used for the outer boundary condition. Convection, wherever it occurs in the models, has been treated as being adiabatic. Masses of 2, 3, 5, 7, 10, 15, 30, and 60 $M_\odot$ have been selected, in conjunction with three different sets of initial hydrogen (X) and metals (Z) abundances:

$$X_e = 0.71\,, \quad Z_e = 0.04\,;$$
$$X_e = 0.73\,, \quad Z_e = 0.02\,;$$
$$X_e = 0.49\,, \quad Z_e = 0.02\,.$$

To obtain the apsidal motion constant k_2, the standard Radau equation has been integrated over the stellar model, following Schwarzschild (1958).

Results for the various stellar models are given in table 1. For each chemical composition, log k_2 attains a maximum value at a mass between 7 and 10 $M_\odot$, at which mass the central condensation of the star is least. Since stellar models constructed with Cox-Stewart opacities are least centrally condensed at $\sim 50\, M_\odot$ (Ezer and Cameron 1967), apsidal motion constants for these models do not show a turnover in the interesting range of masses. The reason for the turnover, generally, is that radiation pressure, which is more important in stars of higher mass, tends to raise the central condensation.[1] In the present instance, the

[1] The effect is apparent in the values of $\rho_c/\langle\rho\rangle$ calculated for stellar models of fixed mass with and without radiation pressure by Auman and Bahng (1965). We note that, crudely, $k_2 \propto (\rho_c/\langle\rho\rangle)^{-1}$ for homogeneous stellar models.

TABLE 1

THEORETICAL MODELS FOR HOMOGENEOUS MAIN-SEQUENCE STARS CONSTRUCTED WITH THE CARSON OPACITIES

Composition	$M/M_\odot$ 2	3	5	7	10	15	30	60
$X_e = 0.71$, $Z_e = 0.04$:								
log $(L/L_\odot)$	1.114	1.795	2.627	3.154	3.684	4.236	5.040	5.684
log T_e	3.943	4.057	4.194	4.278	4.357	4.422	4.519	4.579
log k_2	−2.28	−2.20	−2.13	−2.11	−2.12	−2.18	−2.35	−2.52
$X_e = 0.73$, $Z_e = 0.02$:								
log $(L/L_\odot)$	1.200	1.877	2.692	3.198	3.708	4.244	5.037	5.679
log T_e	3.987	4.100	4.232	4.312	4.388	4.454	4.555	4.622
log k_2	−2.21	−2.13	−2.05	−2.02	−2.00	−2.01	−2.09	−2.20
$X_e = 0.49$, $Z_e = 0.02$:								
log $(L/L_\odot)$	1.713	2.377	3.159	3.640	4.115	4.605	5.310	5.884
log T_e	4.098	4.208	4.333	4.406	4.473	4.529	4.601	4.642
log k_2	−2.21	−2.14	−2.08	−2.05	−2.06	−2.13	−2.31	−2.51

central condensation is further increased by the relatively high opacities obtained by Carson for the low-density conditions encountered in the envelopes of stars more massive than $\sim 7\ M_\odot$. The opacity remains well above the electron-scattering limit in these model envelopes.

If the stellar models are set into uniform rotation, the central condensation will be increased even more. By using the basic equations for a uniformly rotating star reduced to the same form as for a spherical star (Faulkner, Roxburgh, and Strittmatter 1968), we find little change in the stellar models for any realistic value of the surface rotation parameter $\lambda = \frac{2}{3}v^2R/GM$. Our results for the change in the apsidal motion constant due to the change in structure of the star may be expressed as $\Delta \log k_2 = -0.7\lambda$, almost independently of the mass, chemical composition, and opacity representation. Even at critical rotation for breakup at the equator ($\lambda = 0.30$), log k_2 is reduced by only 0.2.

III. COMPARISON OF THEORY AND OBSERVATIONS

The theoretical values of log k_2 for homogeneous main-sequence stars are compared with the empirical values of log k_2 for the four most accurately observed binary systems, as selected by Petty (1973), in figure 1. The observational points for Y Cyg, AG Per, and CO Lac refer to the averages for the two components of each system, as given by Petty; the point for α Vir refers to the primary star alone, as discussed below. To a limited extent, the Carson opacities produce better agreement between theoretical and observed values of log k_2 than do the Cox-Stewart opacities, particularly if Z_e is high. However, the theoretical values are still systematically too high, for any reasonable metals abundance. This is underlined by two facts. First, both spectroscopic evidence and the empirical mass-luminosity relation indicate $X_e = 0.70 \pm 0.03$ and $Z_e = 0.03 \pm 0.01$, on the average, for young stars (e.g., Stothers 1973*b*). Second, the "Thomas-Fermi" opacities of Carson and the "hydrogenic" opacities of Cox and Stewart are likely to bracket the "true" opacities (Carson 1974).

This implies that we must seek an explanation for the discrepancy in terms either of observational errors or of evolutionary effects. The observational errors are very difficult to assess quantitatively, partly because the empirical values of k_2 must usually be expressed (as here) as the average for the two components. For the four systems in question, Mathis (1967) has estimated a typical error of ± 0.2 in log k_2. Such an error would make the discrepancy disappear, but only if it were systematically in the same sense for three of the four systems.

Realistically, we are left only with an evolutionary interpretation. Use of the "hydrogenic" opacities does not yield a low enough value of k_2 during the phase of core hydrogen burning to explain all the observed values, as previous authors have noted. As a desperate remedy, Petty (1973) has suggested that some of the systems are in the pre–main-sequence phase of evolution. On time-scale arguments, however, this is very unlikely, and we prefer an explanation based on the phase of core hydrogen burning. Since, for the stellar masses of interest here, the surface parameters (luminosity, radius, and effective temperature) are far less affected by the atomic model adopted for the opacity than is k_2, we shall concentrate mostly on the differences found for k_2.

a) *Y Cygni*

The nearly identical components ($\sim 17\ M_\odot$ each) of this system are essentially unevolved, according to their luminosities, radii, and effective temperatures (Stothers 1973*a*; Popper 1974). Moreover, Roman (1956) and Olson (1968) have given the luminosity class as V rather than the older value of IV (Morgan, Code, and Whitford 1955). The observed surface rotational parameter λ is about 0.02 for both components; strongly nonuniform rotation is unlikely to exist in

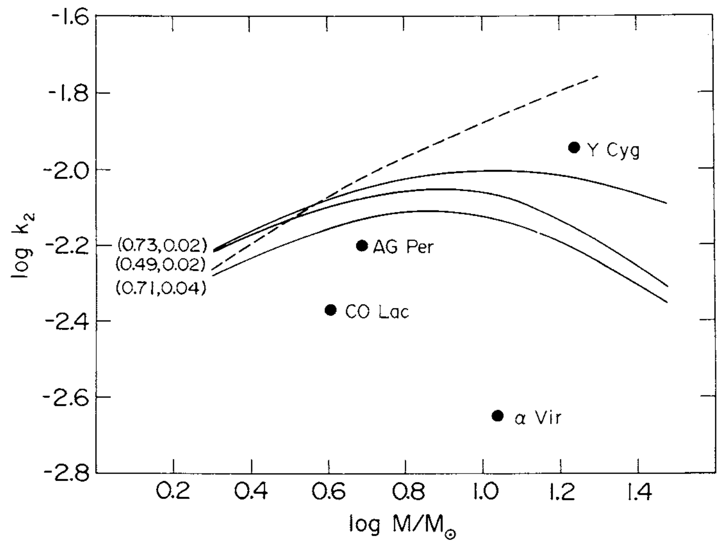

FIG. 1.—The logarithm of the apsidal motion constant is plotted against the logarithm of the stellar mass. Solid lines refer to models of homogeneous main-sequence stars constructed with the Carson opacities for the composition mixtures (X_e, Z_e) as labeled. The dashed line refers to the Ezer-Cameron models constructed with Cox-Stewart opacities for (X_e, Z_e) = (0.739, 0.021), as given by Petty.

such a close detached system (Stothers 1973*a*), and therefore the effect of rotation on k_2 should be negligible. The observed value of $\log k_2 = -1.94$ is bracketed by the values predicted for unevolved stars on the basis of the Carson and Cox-Stewart opacities. However, the observational correction for rotational distortion seems rather large; with no correction at all, $\log k_2 = -2.10$. Possibly the Carson opacities produce better agreement with the observed k_2.

b) α *Virginis*

Mathis and Odell (1973) and Odell (1974) have extensively discussed this detached system on the basis of stellar models constructed with the Cox-Stewart opacities. Odell (1974) concludes that (1) an increased mass or initial helium content is needed to match the present temperature and luminosity of the primary, although the predicted apsidal-motion constant is too large; (2) models with an arbitrarily increased envelope opacity can match the observed apsidal-motion constant, although the need for a larger mass or initial helium content is not eliminated; (3) arbitrary nonconvective mixing out to the boundary of the zero-age convective core can satisfy all the observational requirements; and (4) the primary is probably near or just beyond the stage of hydrogen exhaustion in the core. Admittedly, the first three conclusions are not very pleasant. We shall show that none of the four, with the exception of the need for a higher mass, are required by the observational data if the Carson opacities are adopted.

We adopt the same observational parameters as those used by Odell, with the exception of a small allowance for the partial evolution of the secondary off the zero-age main sequence. Thus the primary is here found to have $\log L/L_\odot = 4.23 \pm 0.05$ and $\log T_e = 4.365 \pm 0.015$, in order to have compatibility with the observed radius of $8.1 \pm 0.5\ R_\odot$ (Herbison-Evans *et al.* 1971). The mass of the primary is $10.9 \pm 1.3\ M_\odot$ (Herbison-Evans *et al.* 1971; Shobbrook, Lomb, and Herbison-Evans 1972). Axial rotation is neglected here because $\lambda = 0.07$ (Stothers 1973*a*).

Following the procedure of Odell (1974), we have computed "normal" evolutionary tracks for a star of $10.9\ M_\odot$ with different (X_e, Z_e) mixtures, until the central hydrogen abundance falls below $X_c = 0.05$. The tracks are shown in figure 2 for four composition mixtures, with lines of constant $\log k_2$ indicated. For (X_e, Z_e) = (0.70, 0.03), the total range of $\log k_2$ during core hydrogen burning is -2.1 to -2.8 in the case of the Carson opacities. Odell obtained $\log k_2 = -1.9$ to -2.3 for the Cox-Stewart opacities. In figure 2, the position of the primary of α Vir is indicated.

We clearly infer, with Odell, that, if $Z_e = 0.03$, the primary has $X_e \approx 0.60$. (The close agreement is due to the accident that our allowance for a brighter secondary is counteracted by Odell's placing the primary at a later stage of evolution.) However, unlike Odell, we can match the predicted and observed apsidal-motion constants for the primary ($\log k_2 \approx -2.65$) by using the Carson opacities. In order to achieve a more realistic hydrogen abundance, such as $X_e \approx 0.70$, we require a mass of $\sim 13\ M_\odot$, but then Z_e should be more like 0.04 in order to fit $\log k_2$. Such a mass is not inconsistent with the available orbital determinations (see table 4 of Shobbrook *et al.* 1972). In either case, the age of the primary is here found to be about 0.8 of its main-sequence lifetime. This is more in line than is Odell's result with the high frequency of β Cephei variables among early B stars (Stothers 1965; Percy 1974)—since we note that the primary of α Vir is a β Cephei variable—and with the observed luminosity class of the primary, which is IV or V (Morgan *et al.* 1955; Lesh 1968; Watson 1972).

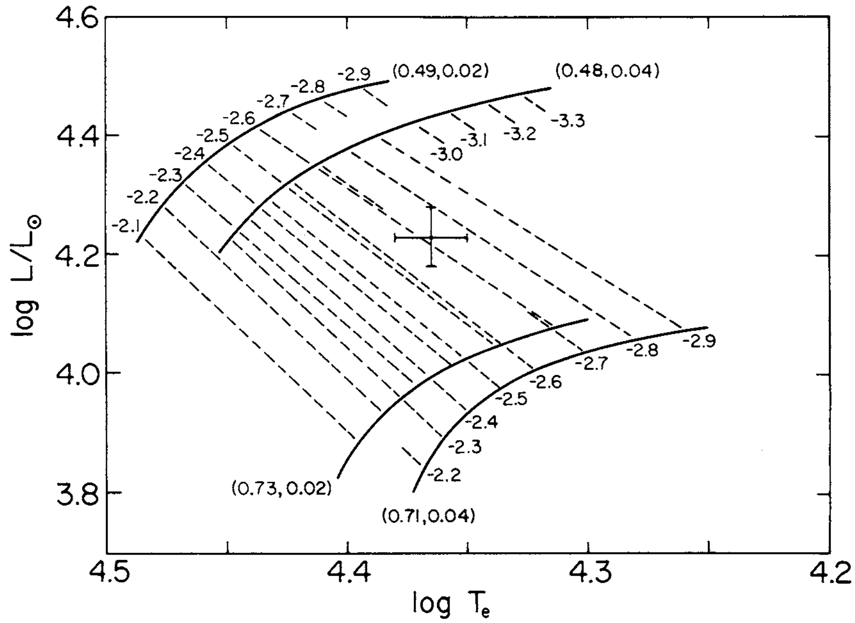

FIG. 2.—Theoretical H-R diagram, showing the evolutionary tracks during core hydrogen burning for a star of 10.9 $M_\odot$ with four different composition mixtures (X_e, Z_e). Values of log k_2 are indicated along the tracks. The cross marks the location of the primary of α Vir.

c) *AG Persei*

The luminosities, radii, and effective temperatures of the rather similar components of this detached system suggest that the stars are slightly evolved (Stothers 1973*a*; Popper 1974), and λ is only 0.02 for each. The masses are $\sim 5\ M_\odot$. Furthermore, the system is a member of the Per OB2 association, whose nuclear age from the main-sequence turnup in the H-R diagram is log τ(years) $= 7.4 - 5Z_e$ (Stothers 1972). For $X_e \approx 0.70$, the main-sequence lifetime of a $5\ M_\odot$ star is log τ(years) $= 7.7 + 5Z_e$. Thus the components of AG Per have gone through about (0.4–$5Z_e$) of their lifetime. Assuming that relative changes of physical quantities along an evolutionary sequence are independent of mass, we predict from table 1 and figure 2 that $\log k_2 = -2.20 - 3(Z_e - 0.03)$, in excellent agreement with the observed value of -2.20. The Cox-Stewart opacities yield a predicted value of log k_2 that is less negative by about 0.10.

d) *CO Lacertae*

Smak (1967) has derived masses of $\sim 4\ M_\odot$ for the rather similar components of this detached system, whose radii and spectral types, when plotted in a theoretical diagram (Stothers 1973*a*), indicate that the stars are partially evolved. Smak, in fact, has assigned luminosity class IV to the (slightly more massive) primary. A $4\ M_\odot$ star with $X_e \approx 0.70$ has a theoretical zero-age main-sequence radius given by $\log R/R_\odot = 0.30 + 2Z_e$, while the total predicted evolutionary shift across the main sequence (Stothers 1972) is $\delta \log R/R_\odot = 0.24$. The observed mean radius of the two components of CO Lac is $\log R/R_\odot = 0.42$. Therefore, the stars have expanded to about (0.5–$8Z_e$) of their maximum radius during core hydrogen burning. From table 1 and figure 2, this implies that $\log k_2 = -2.34 - 7(Z_e - 0.03)$, in excellent agreement with the observed value of -2.37. Again, the Cox-Stewart opacities predict a slightly larger value of log k_2.

e) *CW Cephei*

This system is not shown in figure 1 because of the provisional nature of the apsidal period determined by Nha (1973). The system is detached and has nearly identical components with masses of $\sim 11\ M_\odot$ and λ values of 0.03 (Stothers 1973*a*; Popper 1974). It belongs to the Cep OB3 association, which contains O stars and appears to be essentially unevolved (Garrison 1970). In fact, Popper's (1974) new data for CW Cep indicate that the components are only very slightly evolved. For $X_e \approx 0.70$, we predict theoretically that $\log k_2 = -2.07 - 6(Z_e - 0.03)$, while, for the Cox-Stewart opacities, log k_2 would be less negative by about 0.17. By assuming synchronous rotation of the components (Stothers 1973*a*) and by adopting the new observational data of Popper and Nha, we find a *provisional* empirical value of $\log k_2 \approx -2.00$. Exclusive use of Nha's (unpublished) own data yields $\log k_2 \approx -2.14$.

IV. CONCLUSIONS

The apsidal-motion constants, k_2, for models of homogeneous main-sequence stars have been computed by employing the new "Thomas-Fermi" opacities of Carson. These models are similar to those constructed with the "hydrogenic" opacities of Cox and Stewart up to $\sim 7\ M_\odot$, but yield significantly smaller values of k_2 at higher masses. Nevertheless, the predicted apsidal-motion constants are still systematically too large compared with most of the observed constants, for any realistic chemical composition. Uniform rotation is found to lower these constants only slightly. The effect of "normal" evolution during core hydrogen burning is calculated, and is found to produce good detailed agreement between the predicted and observed apsidal motion constants. The anomalous evolution suggested to exist for the primary of α Vir (Mathis and Odell 1973; Odell 1974) is not strictly required by the available observational data if Carson's opacities are adopted. Thus the discrepancy between the physical and the astronomically inferred opacity—a problem which has persisted in various forms since the original investigations of radiative stars by Eddington—may now have disappeared to a large extent.

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